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ABSTRACT

The purpose of this module is to acquaint students with the terms prime, composite, and factor. This is done by offering a definition of each term, then reinforcing its meaning through activities. (Author/MK)

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TOPICAL MODULE FOR USE

IN A

MATHEMATICS LABORATORY SETTING

PRIME NUMBERS

by

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Edited by
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TEACHER'S GUIDE

PRIME NUMBERS

This module may be used in a variety of ways.

1. Groups of students may be assigned to various experiments according to past performance.
2. The class may be divided so each group has a group leader who has the ability to help the other students.
3. Teachers may use a group-method of their own choosing or the entire class.

Materials

.Teacher must provide:

1. 12 cubes for dice: (A.S. #1 - 8 cubes), (A.S. #3 - 1 cube), (A.S. #5 - 3 cubes).
2. 48 colored chips. Suggest you use 4 different colors with 12 chips of each color. (A.S. #1 - 48 chips), (A.S. #5 - 48 chips).
3. 174 cards: (A.S. #7 - 101 of one color and 73 of another color.)

Time Schedule

.During field testing, this module took an average of _____ fifty-minute class periods to complete.

OBJECTIVES

.The student will be able to factor any natural number into integers.

.The student will be able to distinguish between prime and composite numbers.

.The student will be able to write prime factors for composite numbers.

OVERVIEW

.The purpose of this module is to acquaint students with the terms prime, composite, and factor. This is done by offering a definition of each term, then reinforcing its meaning through activities.

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I. Activity Sheet #1

A. Materials needed:

1. Eight number cubes, each numbered 1-6.
2. A supply of 48 plastic square chips
3. Student Worksheet #1

B. Teaching suggestions:

1. The purpose of the first activity is to give the students an opportunity to practice finding factors of numbers. Ordinary number cubes, numbered 1-6 will generate numbers 8-48.
2. The game may be altered by the addition of a "challenge" aspect by an opponent if a combination is not seen by the player.

C. Solutions:

8	1,8; 2,4
9	1,9; 3,3
10	1,10; 2,5
11	1,11
12	1,12; 2,6; 3,4
13	1,13
14	1,14; 2,7
15	1,15; 3,5
16	1,16; 2,8; 4,4
17	1,17
18	1,18; 2,9; 3,6
19	1,19
20	1,20; 2,10; 4,5

21	1,21; 3,7
22	1,22; 2,11
23	1,23
24	1,24; 2,12; 3,8; 4,6
25	1,25; 5,5
26	1,26; 2,13
27	1,27; 3,9
28	1,28; 2,14; 4,7
29	1,29
30	1,30; 2,15; 3,10; 5,6
31	1,31
32	1,32; 2,16; 4,8
33	1,33; 3,11
34	1,34; 2,17
35	1,35; 5,7
36	1,36; 2,18; 3,12; 4,9; 6,6
37	1,37
38	1,38; 2,19
39	1,39; 3,13
40	1,40; 2,20; 4,10; 5,8
41	1,41
42	1,42; 2,21; 3,14; 6,7
43	1,43
44	1,44; 2,22; 4,11
45	1,45; 3,15; 5,9
46	1,46; 2,23
47	1,47
48	1,48; 2,24; 3,16; 4,12; 6,8

II. Activity Sheet #2

A. Materials needed:

1. Student Worksheet #1

B. Teaching suggestions:

1. This exercise makes reference to the game on Activity #1. It may be helpful

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for the students to have their score sheets (Student Worksheet #1) available.

2. A class discussion of the terms "prime" and "composite" is advised.

C. Solutions:

1. Prime possibilities:
11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.

2. All others are composite.

3. The prime numbers resulted in only one rectangle; i.e., 1 x N.

4. More points were scored if the sum was composite.

III. Activity Sheet #3

A. Materials needed:

1. A number cube marked 2 to 7.

2. A paper and pencil for scoring.

B. Teaching suggestions:

1. Discuss the term "factor".

2. The students should be encouraged to develop a strategy for scoring points.

C. Solutions:

1. The number called should be a multiple of 2 to 7.

2. Primes or multiples of primes greater than 7 will never score points.

3. The number 420 is the least common multiple of 2 to 7 and will score a point every time.

4. A variation of this would be to alter the numbers on the cube and find the LCM.

IV. Activity Sheet #4

A. Materials needed:

1. Student Worksheet #'s 4 - 4B

2. Felt marking pen.

B. Teaching suggestions:

1. The graph in this exercise is formed by the Cartesian product of the numbers 2 to 41 and their prime factors; i.e., number, prime factors.

2. It may be best to do the first few together as a group.

3. The terms row and column are defined to be:

row - horizontal
column - vertical

4. This graph provides more information than the Sieve of Erostothenes, but it may help to use both.

C. Solutions:

1. (See appended graph)

2. All of the columns contain a black square.

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3. Only those rows labeled by a prime number have a black square.

4. Rows with 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41

5. Rows without 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39, 40.

6. Yes. The top diagonal contains black squares that identify the primes.

7. For the number thirty, 2, 3, and 5 are marked. The prime factors of 30 are 2, 3, and 5.

8. Prime factors:

$12 = 2 \times 2 \times 3$
$25 = 5 \times 5$
17 prime
$38 = 2 \times 19$
41 prime .

V. Activity Sheet #5

A. Materials needed:

1. Three dice (1) 1,2,3,4,5,6; (2) 2,3,4,5,6,7; (3) 1,9,11,13,17,19

2. Student Worksheet #5 (Game board)

3. Markers - chips, paper clips, etc. (They should be different for each player so that at the end of the game, ownership can be identified.)

B. Teaching suggestions:

1. This game is designed to help students

distinguish between prime and composite numbers.

2. The suggested numbers for marking the cubes may be changed to incorporate the entire board.

3. Anything - paper clips, chips, beans, etc. can be used as markers on the board.

C. Solutions:

1. Note that the only time a prime can be covered is when a 1 appears on one of the die and a prime on another.

2. Solutions will vary from game-to-game.

VI. Activity Sheet #6

A. Materials needed:

1. Student Worksheet #6

B. Teaching suggestions:

1. The notion of forming factor networks or trees can be presented as a group exercise. When forming factor trees, the student needs to check all the primes up to the square root of the number. Insist on this procedure before allowing the student to label a number prime.

2. It is not necessary that a prime be selected first in forming factor networks.

3. The rules may need more explanation, especially divisibility by three.

4. It may be a good time to introduce the term "multiple".

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C. Solutions:

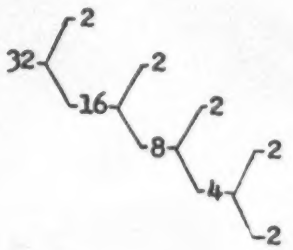
1. E = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30

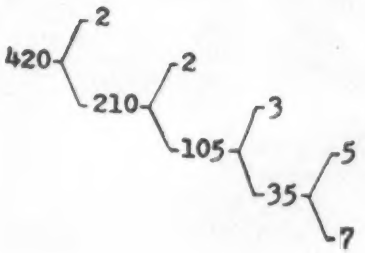
   T = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45

   F = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75

2. All even numbers end with 0, 2, 4, 6, 8

3. 237 is divisible by 3 since 2 + 3 + 7 = 12 and 12 is divisible by 3.

4. 



VII. Activity Sheet #7

A. Materials needed:

1. Deck of PRIME cards and NUMBER cards.

PRIME Deck	Number	Number of cards
	2	12
	3	10
	5	10
	7	7
	11	4
	13	4
	17	3
	19	3
	23	2
	29	2
	31	2

and one each of 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

NUMBER Deck

Card numbers 1 to 99, plus two "Wild Cards" to be used as any prime factor the holder chooses.

If the decks are made up on different colored cards, they will be easier to keep separated. You may want to use different colored marking pens as well.

B. Teaching suggestions:

1. This game is designed to give students practice in finding prime factors of counting numbers.
2. Two or more students can play the game.

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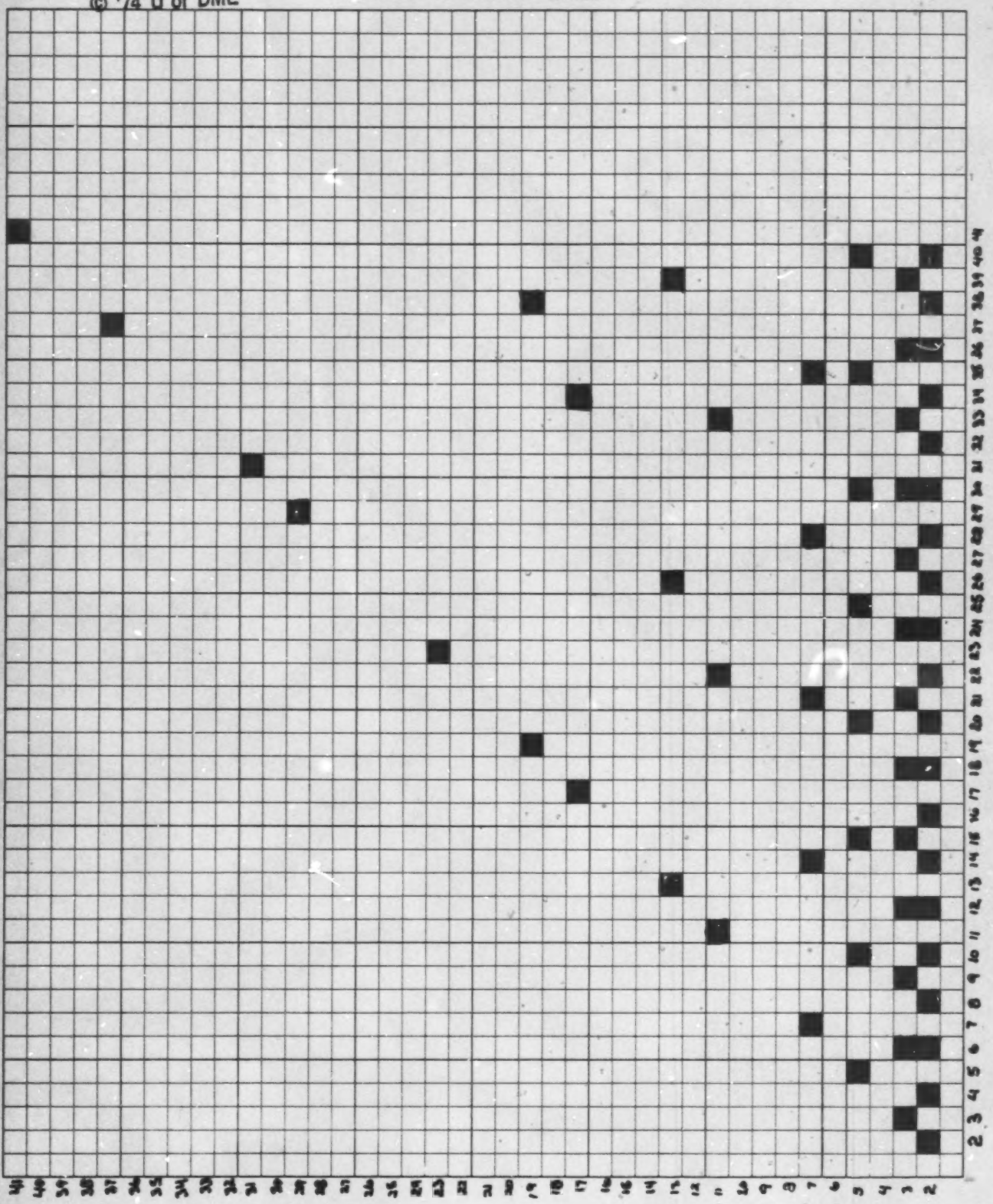
Answers to Student Worksheet #4



SOLUTIONS FOR THE PRETEST AND POSTTEST

Pretest

1. b
2. b
3. d
4. 1, 2, 3, 5, 6 and 30
5. 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72
6. 1, 2, 7 and 14
7. 1, 7, 13 and 91
8. 1, 2, 3, 4, 6, 8, 12 and 24.

Posttest

1. 3 x 3 x 3
2. 5 x 17
3. Prime
4. 2 x 2 x 5 x 13
5. 2 x 3 x 41
6. Prime
7. 2 x 2 x 7 x 7 x 19
8. 11 x 101
9. 2 x 2 x 2 x 2 x 2 x 2 x 2
10. 71 x 73 .

PRETEST

_____ 1. A prime number

a) has only one digit

b) is divisible only by one and itself

c) is divisible by a number other than one and itself

d) is always odd.

_____ 2. A factor

a) can only be prime

b) divides a number evenly

c) is always even

d) divides every number.

_____ 3. A composite number

a) can be prime

b) has more than one digit

c) cannot be odd

d) is divisible by numbers other than one and the number itself.

Find at least four numbers that will divide evenly into each number listed.

4. _____ 30

5. _____ 72

6. _____ 14

7. _____ 91

8. Find all the numbers that will divide evenly into 24.

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CHIPS'N RECTANGLES

Number of players: Two or more

Materials: Eight dice labeled 1 to 6
Forty-eight chips
Student Worksheet #1

Object of the game: Make as many rectangles as possible with the chips.

Rules:

Each player rolls two dice, and the person with the greatest sum begins. The first player rolls the eight dice and determines the sum. He selects the number of chips corresponding to the sum and begins making a rectangle using all the chips. He records the dimensions of the rectangle on the score sheet and attempts to make another rectangle using all the chips. The player scores a point for every rectangle. Each player is allowed five minutes for his turn.

The winner is determined by the player who has the most points after three rounds of play.

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Player's Name	Sum of the Dice (No. of Chips)	Dimensions of the Rectangle	Points
First turn			
Second turn			
Third turn			
Total			
First turn			
Second turn			
Third turn			
Total			
First turn			
Second turn			
Third turn			
Total			
First turn			
Second turn			
Third turn			
Total			
First turn			
Second turn			
Third turn			
Total			

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The word <u>number</u> in this module refers to the positive whole numbers: (1, 2, 3, 4, ...)

A <u>composite</u> number is one which can be written as a product of two whole numbers other than 1 and itself. Example: Six is composite since $6 = 2 \times 3$.

<u>A prime</u> number is one which can be written <u>only</u> as a product of 1 and itself. Example: Five is prime since it can be written only as a product of 1 and 5; i.e., $5 = 1 \times 5$.

Examine the score sheet on Student Worksheet #1. Which numbers were prime? Which numbers were composite? What method did you use for determining if a number was prime or composite? Can you score more points if the sum of the dice is prime or if it is composite?

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A factor is a whole number which divides evenly into another number.
Example: 1, 2, 3, 6 are factors of 6.

FACTORS PLUS

Number of players: Two or more

Materials: A die labeled 2 through 7

Object of the game: Call a number which has as a factor the number turned up on the die.

Rules:

Each player rolls the die and the player with the highest number begins. The first player begins by calling a number and then rolls the die. If the number turned up on the die is a factor of the number called, he scores a point. Proceed clockwise until every player has had ten opportunities to roll the die. A point is scored whenever a factor turns up. The player with the greatest number of points is the winner.

Did you find that certain numbers called were more apt to score points than others?

What happened if you called the number 11 or 17? What were your chances of scoring points? What about the number 420?

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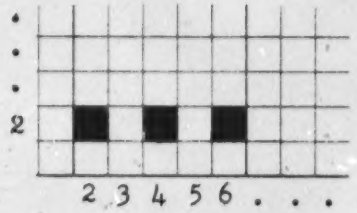
In order to determine which numbers are prime and which numbers are composite, it is best to go about the search in a systematic way. Look at Student Worksheet #4. Consider the numbers along the bottom (horizontal) as the numbers we want to examine. The numbers along the side (vertical) are the factors.

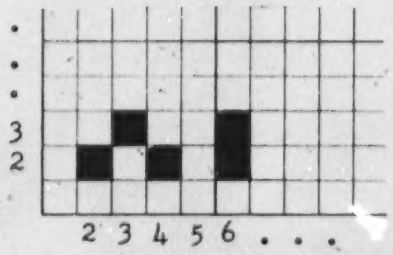
Find the number 2 along the bottom (horizontal). Shade the square where the column and row containing the number 2 intersect. Shade all the squares in the row labeled with the number 2 that have number 2 as a factor.

Example:



Go to the next column that does not have any shaded squares (the column labeled with the number 3, in this case.) Shade the square where the column and row labeled with the number 3 intersect. Shade all the squares in the row labeled with the number 3 that have 3 as a factor.

Example:



Go to the next column that does not have any shaded squares (the column labeled with the number 5, in this case.) Shade the square where the column and row labeled with the number 5 intersect. Shade all the squares in the row labeled with the number 5 that have 5 as a factor.

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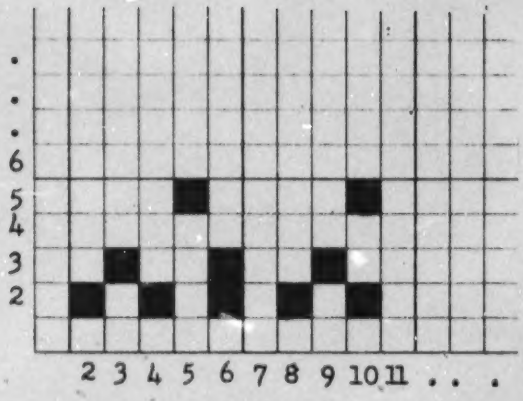
Example:



Continue with the above process until Student Worksheet #4 is completed.

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Student Worksheet #4

Examine the chart on Student Worksheet #4. Do all the columns have a black square? (Columns are vertical.) Do all the rows have a black square? (Rows are horizontal.)

Each row is labeled with a number, 1-41. Make a list on Student Worksheet #4B of the numbers representing rows with a black square, and a list of the numbers representing rows which do not contain black squares.

Is it possible to find the prime numbers by looking at a diagonal?

Look at the column labeled 30 on Student Worksheet #4. Which squares are black? What are the prime factors of 30?

Complete Student Worksheet #4B.

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Rows with at least one black square

Rows without a black square

Can you describe a method for finding prime numbers from the chart?

What are the prime factors of 30?

Find the prime factors of each of the following.

1. 12 _____
2. 25 _____
3. 17 _____
4. 38 _____
5. 41 _____

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ROLL AND FACTOR

Number of players: Two or more

Materials: A distinct color of chips for each player
Student Worksheet #5 (Game board)
Three dice marked: (1) 1, 2, 3, 4, 5, 6; (2) 2, 3, 4, 5, 6, 7;
(3) 1, 9, 11, 13, 17, 19

Object of the game: Cover as many squares on the game board as possible.

Rules:

Determine the first player by the person who rolls the highest sum on the dice, then continue play in a clockwise manner. Each player needs to have a supply of a certain color of chips to identify the squares which belong to him. Player one rolls the dice, then tries to form as many products as he possibly can, using the numbers on the dice. He places a chip on each product on the game board; if he is able to cover a prime, he may place three chips on that square. The player with the most chips on the board after each person has had five turns is the winner. Once a square is covered, it cannot be covered again.

Example: Die faces are 2, 4, 9 — $2 \times 4 = 8$, $2 \times 9 = 18$, $4 \times 9 = 36$, $2 \times 4 \times 9 = 72$. Cover four squares, score four points.

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1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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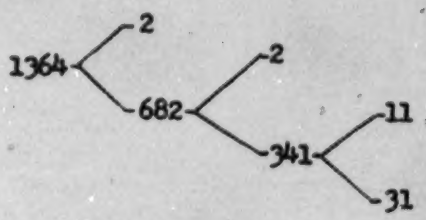
In order to perform certain mathematical operations, it is sometimes helpful to know the prime factors of composite numbers. By dividing prime numbers into the given number, we can arrive at the prime factors. This can be done by beginning with the smallest prime factor and forming a network or tree.

For example:



Factors are: 2, 2, 11, 31

Three rules are helpful in finding prime factors:

1. Two is a factor of all even numbers.
2. If the sum of the digits of a number are divisible by three, three is a factor of the number. (582 is divisible by 3 since 5 + 8 + 2 = 15 and 15 is divisible by 3.)
3. Five is a factor of all numbers ending in five or zero.

Complete Student Worksheet #6.

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List the first 15 elements of the sets E, T and F where
E = even numbers , T = numbers divisible by three ,
F = numbers divisible by five .

E =

T =

F =

List the set of digits common to all even numbers.

Is 237 divisible by 3? How can you decide, without dividing three into 237?

Factor each of the following into primes, using factor trees.

32

420

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PRIME PILES

Number of players: Two or more

Materials: Deck of NUMBER cards
Deck of PRIME cards

Object of the game: Collect five piles of prime factors for a number.

Rules:

Deal seven cards to each player from the PRIME deck. Dealer then turns over five cards from the NUMBER deck in the middle of the table. The player to the left of the dealer begins play by placing one of his cards next to a number on the table. The card must be a prime factor of the number. He then draws a card from the PRIME deck to complete his play. If he has no prime factors for the numbers on the board, he must "pass". The player who puts down the last prime factor for a number collects the pile of cards (the NUMBER cards and PRIME cards) and places them in front of him. He then turns over another NUMBER card and places it in the middle of the table.

The first player to accumulate three stacks is the winner.

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POSTTEST

Find the prime factors of the following numbers. Write your answer as a product of prime factors.

1. _____ 27

2. _____ 85

3. _____ 83

4. _____ 260

5. _____ 246

6. _____ 97

7. _____ 3724

8. _____ 1111

9. _____ 128

10. _____ 5183

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